



In this example, we have an irregular pentagon ($N=5$) with vertices numbered 0 through 4 ($N-1$). The small arc attached to each vertex k is labeled r_k , and the distance between any two vertices j, k is $d_{j,k}$. Alternatively, each vertex's small radius can be labeled according to its sister vertex, which is the vertex $(N-1)/2$ in the clockwise direction from itself. For example, vertex 2 is the sister vertex of vertex 0, and so it can be labeled either r_2 or a_0 . The diameter of the shape, D , is then equivalent to $r_k + a_k + d_{k,s}$, where k is any vertex ($0 \dots N-1$), and s is k 's sister vertex: $s = (k + (N-1)/2) \% N$.

The resulting N equations can be expressed as a matrix equation:

$$\mathbf{R} + \mathbf{A} + \mathbf{D} = \Phi \quad ,$$

where \mathbf{R} , \mathbf{A} , and \mathbf{D} are $N \times 1$ matrices of the small radius, the sister vertex small radius, and the vertex-to-sister distance of each vertex in the polygon. Φ is an $N \times 1$ matrix where every element is the desired final diameter of the shape of constant width.

The a_k are just different, shifted numberings of the r_k , and this can be expressed using a shift matrix \mathbf{J} defined by its elements j_{ik} :

$$j_{ik} = \begin{cases} 1 & \text{if } k = (i + \frac{N-1}{2}) \% N \\ 0 & \text{otherwise} \end{cases} .$$

$$\mathbf{R} + \mathbf{J} \mathbf{R} + \mathbf{D} = \Phi$$

We therefore have the following equation for \mathbf{R} (and $N \times N$ identity matrix \mathbf{I}):

$$\mathbf{R} = (\mathbf{J} + \mathbf{I})^{-1} (\Phi - \mathbf{D}) .$$

From this, we know the small radius for each vertex. And since we know the distance between all vertices (since we know their coordinates), we can compute the major radii as $D - r_k$. This is everything needed to draw the shape of constant width.